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Analysis and Validation of Linear Half Car Passive Suspension System with Different Road Profiles

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Abstract: *In this paper the study of linear passive suspension system of half car passive suspensions system of vehicle is carried out. The motive of this study is to find out the behavior of car for different types of road profiles. The 4-DOF mathematical model is established and based on this model using Newton's 2nd law of motion the MATLAB-SIMULINK model is developed. The simulation model is also validated with an analytical State Space approach. This validated half car SIMULINK model is also used to analyze RMS acceleration, i.e. Ride Comfort for different input road profiles.*

Keywords – *1/2 car, Linear, Passive Suspension, SIMULINK Model, Validation*

I. Introduction

From the last few decades the automobile suspension system analysis is become one of the leading topics for the study because of customer comfort, safety and overall performance of vehicle. The 1/2 car model is useful to observe the changing response of different conditions of roads. The sprung mass, unsprung mass, spring stiffness, damping coefficients of suspensions, distance from center of gravity from wheels and tire stiffness are considered to analyze the various responses.

In this paper the 1/2 car model is considered for more precise analysis compared to the quarter car model. A 4-DOF half car mathematical model is developed by using the Newton's 2nd law of motions. From these equations a MATLAB-SIMULINK model is produced. Same equations are solved by using analytical method i.e. State Space approach. By comparing their displacement graphs the simulation and analytical model is validated. The validated simulation model is also used to find RMS acceleration for different types of road input profiles such as step bump, half sine bump, and random input road profile.

There is a significant development in the analysis of the mathematical modeling tools. Most studies of the vehicle flow depend on the supposition of various parameters of the system. But in actual practice the values may vary too much and also different tolerances and friction taking into the considerations. In cars, buses and trucks there are different arrangements in the design and also different size mass and performance. Because of such variations the problems of vehicle vibrations also became a good research topic and have a great significance in the realistic engineering applications [1].

Tejas P. Turakhia, Prof. M. J. Modi have studied the types of suspension system i.e. Semi-active, Passive and Active suspension. The costs of semi-active and active suspensions are quiet high which required external energy and actuator. The passive suspension system is most commonly used system. The authors also developed the 2 Degree of Freedoms System and analyzed with the help of MATLAB SIMULINK and state space. For the practical purpose the half car model is used for study of the changing responses between vehicle and road profiles. The suspension system is important to amend the ease of ride and handling of road in the automobiles vehicles [2].

Anirban C. Mitra, Nilotpal Benerjee have suggested that the half car model is more accurate approach because of pitch. From the last few decades the mathematical modeling is the important tool for the analysis of the vehicle dynamics and also concluded that it has a lot of potential. In the most of cases the constant values of stiffness and coefficient of damping are taken into the consideration. In the vehicles, different variables like mass of vehicles, positions of passengers can have huge impact. Also, same type of vehicles having different mass, size and layout. Quarter car model only have vertical motion but half car model can be used vertical as well as pitch movement [3].

In this paper the 1/2 car model is considered for more precise analysis compared to the quarter car model. A 4-DOF half car mathematical model is developed by using the Newton's 2nd law of motions. From these equations a MATLAB-SIMULINK model is produced. Same equations are solved by using analytical method i.e. State Space approach. By comparing their displacement graphs the simulation and analytical model is validated. The validated simulation model is also used to find RMS acceleration for different types of road input profiles such as step bump, half sine bump, and random input road profile.

II. Half Car Passive Suspension System

The arrangement of the half car linear passive suspension system is as shown in Fig.1. The system consists sprung mass M_s, Pitch axis moment of inertia J, unsprung mass of front wheel M_{wf}, unsprung mass of rear wheel M_{wr}, stiffness of front spring K_{sf}, stiffness of rear spring K_{sr}, damping coefficient of front C_{sf}, rear damper damping coefficient C_{sr}, front tire stiffness K_{wf}, rear tire stiffness K_{wr}, the length from center of gravity to wheels are a and b respectively. Z_{cg}, Z_{wf}, Z_{wr}, Z_{rf}, Z_{rr} are the displacement of mass sprung, unsprung and tire displacement respectively. Due the pitch axis M.I. J the system has 4-degrees of freedoms.

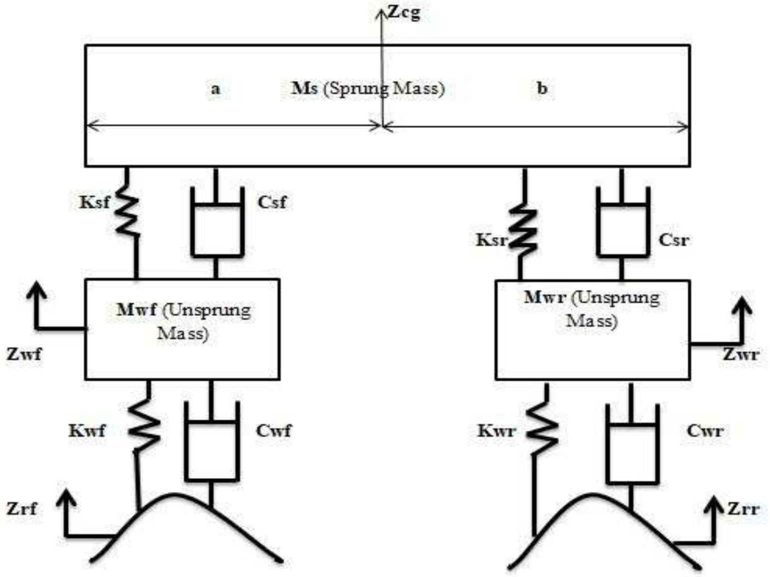

Fig.1 Half car linear passive suspension system

III. MATHEMATICAL MODELING

The mathematical modeling of linear half car suspension system is 4 DOFs. From the above Fig. 1 and using Newton's second law of motion the equations of motions are obtained. The equations of half car linear passive suspension system are

$$\boldsymbol{Ms.\ddot{Z}_{cg}} = -K_{sf}(Z_{cg} - a.\theta) - C_{sf}(\dot{Z}_{cg} - a.\dot{\theta}) + K_{sf}.Z_{wf} + C_{sf}.\dot{Z}_{wf} - K_{sr}(Z_{cg} + b.\theta) - C_{sr}(\dot{Z}_{cg} + b.\dot{\theta}) + K_{sr}.Z_{wr} + C_{sr}.\dot{Z}_{wr}) \quad (1)$$

$$\boldsymbol{J\ddot{\theta}} = (-K_{sf}.a)\left(Z_{cg} + \theta(-a)\right) + C_{sf}.a\left(\dot{Z}_{cg} + \dot{\theta}.(-a)\right) + (-K_{sf}.a.Z_{wf}) + (-C_{sf}.a)(\dot{Z}_{wf}) + (-K_{sr}.b)(Z_{cg} + \theta.b) - C_{sr}.b(\dot{Z}_{cg} + \dot{\theta}.b) + K_{sr}.b.Z_{wr} + C_{sr}.b.\dot{Z}_{wr} \quad (2)$$

$$\boldsymbol{M_{wf}.\ddot{Z}_{wf}} = K_{sf}\left(Z_{cg} + \theta(-a)\right) + C_{sf}\left(\dot{Z}_{cg} + \dot{\theta}(-a)\right) + (-K_{wf}.Z_{wf}) - K_{sf}.Z_{wf} - C_{wf}.\dot{Z}_{wf} - C_{sf}.\dot{Z}_{wf} + K_{wf}.Z_{rf} \quad (3)$$

$$\boldsymbol{M_{wr}.\ddot{Z}_{wr}} = K_{sr}(Z_{cg} + \theta.b) + C_{sr}(\dot{Z}_{cg} + \dot{\theta}.b) - K_{wr}.Z_{wr} - K_{sr}.Z_{wr} - C_{wr}.\dot{Z}_{wr} - C_{sr}.\dot{Z}_{wr} + K_{wr}.Z_{rr} \quad (4)$$

Where $\ddot{Z}_{cg}$, $\dot{Z}_{cg}$, Z_{cg} are the acceleration, velocity and displacement of sprung mass C.G. respectively. $\ddot{\theta}$, $\dot{\theta}$, θ are the angular acceleration, angular velocity, angular displacement respectively. $\ddot{Z}_{wf}$, $\dot{Z}_{wf}$, Z_{wf} are the acceleration, velocity and displacement of front unsprung mass respectively, $\ddot{Z}_{wr}$, $\dot{Z}_{wr}$, Z_{wr} are the acceleration, velocity and displacement of rear unsprung mass respectively. Length from CG to front and rear wheel are a and b respectively.

IV. STATE SPACE APPROACH

To validate the mathematical modeling there are methods i.e. state space approach and MATLAB SIMULINK model. By comparing the both approaches the mathematical model is validated. So for the theoretical approach i.e. state space following calculations are done. From the above equations (1), (2), (3) and (4) the state space equation is obtained. It is supposed that the state space variables are given by the following expressions,

Assume

$X_1 = Z_{cg}$, $X_2 = \dot{Z}_{cg}$, $X_3 = \theta$, $X_4 = \dot{\theta}$

$X_5 = Z_{wf},\qquad X_6 = \dot{Z}_{wf},\qquad X_7 = Z_{wr},\ X_8 = \dot{Z}_{wr}$

State Space Equation is:

$\dot{X} = AX + BU$

$$\begin{bmatrix} dX1 \\ dX2 \\ dX3 \\ dX4 \\ dX5 \\ dX6 \\ dX7 \\ dX8 \end{bmatrix}$$

$$= \left\{\begin{matrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \left[\frac{-K_{sf}-K_{sr}}{Ms}\right] & \left[\frac{-C_{sf}-C_{sr}}{Ms}\right] & \left[\frac{K_{sf}.a-K_{sr}.b}{Ms}\right] & \left[\frac{C_{sf}.a-C_{sr}.b}{Ms}\right] & \left[\frac{K_{sf}}{Ms}\right] & \left[\frac{C_{sf}}{Ms}\right] & \left[\frac{K_{sr}}{Ms}\right] & \left[\frac{C_{sr}}{Ms}\right] \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \left[\frac{-K_{sf}.a-K_{sr}.b}{J}\right] & \left[\frac{C_{sf}.a-C_{sr}.b}{J}\right] & \left[\frac{K_{sf}.a^2-K_{sr}.b^2}{J}\right] & \left[\frac{-C_{sf}.a^2-C_{sr}.b^2}{J}\right] & \left[\frac{-K_{sf}.a}{J}\right] & \left[\frac{-C_{sf}.a}{J}\right] & \left[\frac{K_{sr}.b}{J}\right] & \left[\frac{C_{sr}.b}{J}\right] \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \left[\frac{K_{sf}}{M_{wf}}\right] & \left[\frac{C_{sf}}{M_{wf}}\right] & \left[\frac{-K_{sf}.a}{M_{wf}}\right] & \left[\frac{-C_{sf}.a}{M_{wf}}\right] & \left[\frac{-K_{wf}-K_{sf}}{M_{wf}}\right] & \left[\frac{-C_{wf}-C_{sf}}{M_{wf}}\right] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \left[\frac{K_{sr}}{M_{wr}}\right] & \left[\frac{C_{sr}}{M_{wr}}\right] & \left[\frac{K_{sr}.b}{M_{wr}}\right] & \left[\frac{C_{sr}.b}{M_{wr}}\right] & 0 & 0 & \left[\frac{-K_{wr}-K_{sr}}{M_{wr}}\right] & \left[\frac{-C_{wr}-C_{sr}}{M_{wr}}\right] \end{matrix}\right\} \begin{Bmatrix} X1 \\ X2 \\ X3 \\ X4 \\ X5 \\ X6 \\ X7 \\ X8 \end{Bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_{wf}}{M_{wf}} & 0 \\ 0 & 0 \\ 0 & \frac{K_{wr}}{M_{wr}} \end{bmatrix} . \begin{bmatrix} Z_{sf} \\ Z_{sr} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$Y = CX + DU$

$$\begin{bmatrix} Y1 \\ Y2 \\ Y3 \\ Y4 \\ Y5 \\ Y6 \\ Y7 \\ Y8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} . \begin{bmatrix} X1 \\ X2 \\ X3 \\ X4 \\ X5 \\ X6 \\ X7 \\ X8 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} . \begin{bmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \\ U8 \end{bmatrix}$$

Where A is Matrix of State, B and C are i/p and o/p Matrix and D is matrix of Direct Transmission.

Table 1 gives the variables for the 1/2 car model which is useful to the MATLAB Simulink and state space calculations to validate the mathematical modeling.

Table 1: Parameters for Half Car Linear Suspension System

Suspension Parameters	**Symbol**	**Unit**	**Value**
Sprung Mass	Ms	kg	600
Pitch Axis Moment of Inertia	J	kg-m^2	730
Unsprung (Front)Mass	Mwf	kg	45
Unsprung (Rear) Mass	Mwr	kg	45
Spring (front)Stiffness	Ksf	N/m	18000
Spring (rear) Stiffness	Ksr	N/m	18000
Damping (Front) Coefficient	Csf	N-s/m	500
Damping (Rear) Coefficient	Csr	N-s/m	500
Tire (front) Stiffness	Kwf	N/m	102017.2
Tire (rear) Stiffness	Kwr	N/m	102017.2
Tire (front) Damping Coefficient	Cwf	N-s/m	138
Tire (rear) Damping Coefficient	Cwr	N-s/m	138
Length CG to wheel (front)	A	M	1.5
Length CG to wheel (rear)	B	M	1.15

V. MATLAB SIMULINK MODEL

The Fig. 2 shows the MATLAB SIMULINK model which is developed by using mathematical modeling. In the MATLAB SIMULINK the road input is considered as a step input. SIMULINK model is the block diagram of the mathematical model which gives the amplitude v/s time graph. By using graph it can be calculated that displacement of sprung as well as unsprung mass. And also the acceleration and velocity of sprung and unsprung mass can be calculated.

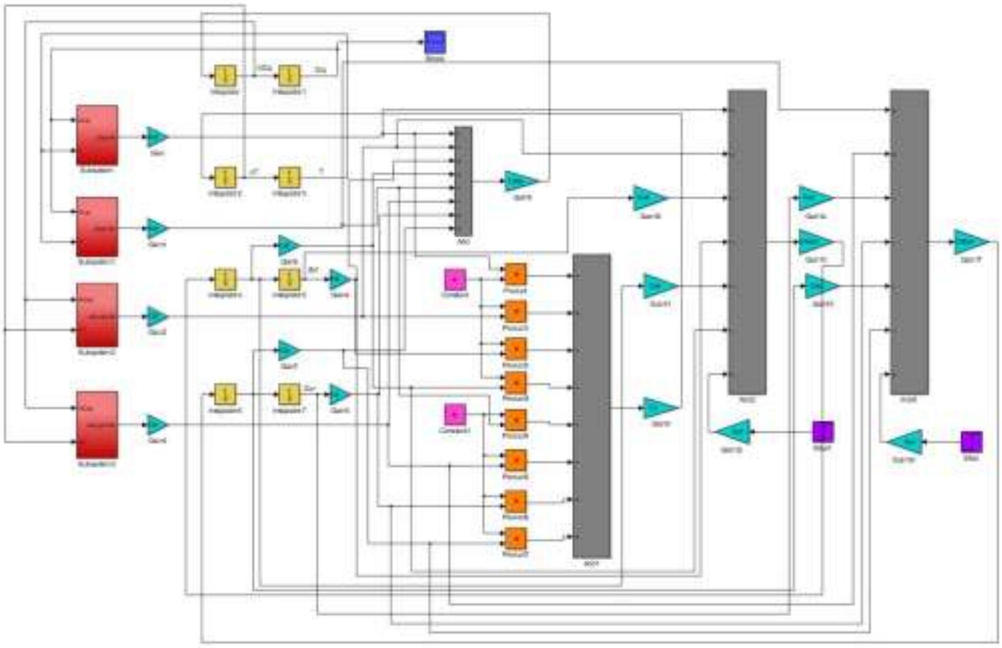

Fig.2: MATLAB SIMULINK of linear passive half car suspension system

VI. VALIDATION

With the help of state space approach and MATLAB SIMULINK the mathematical model is validated. The sprung mass displacement of Analytical Solution using MATLAB Coding is as shown in Fig. 3 and sprung mass displacement of MATLAB SIMULINK model is as shown in Fig. 4. The displacement is same in the state space and MATLAB Simulink, so it can be validated using the graphs.

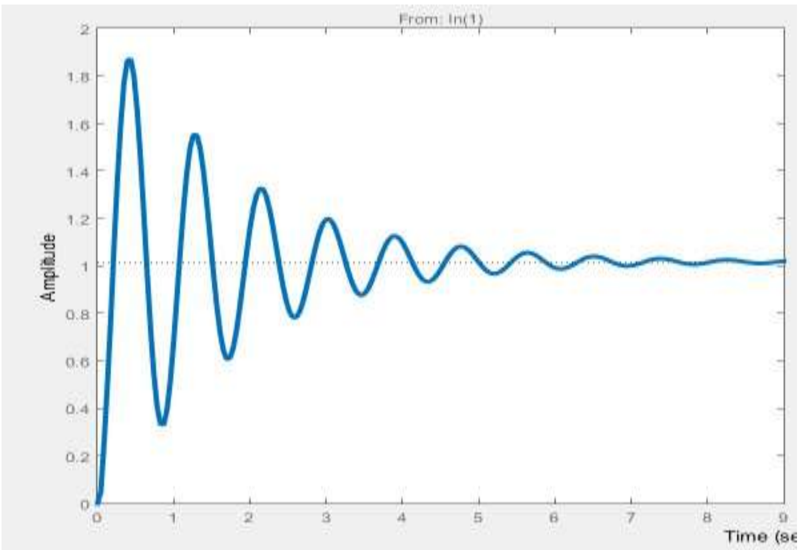

Fig.3: Sprung mass displacement graph by analytical solution using MATLAB coding

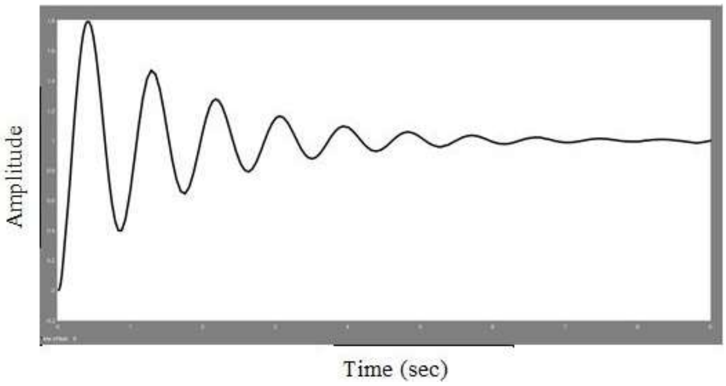

Fig.4: Sprung mass displacement graph MATLAB SIMULINK

VII. RESULTS AND DISCUSSIONS

The half car mathematical model is validated using the displacements graphs as shown in Fig.3 and Fig.4. The purpose of the development of 1/2 car model Simulink is to find out the RMS acceleration. The RMS acceleration is inversely proportional to the ride comfort. To find the RMS acceleration the different types of road input is considered such as step bump, sine wave, half sin bump, random input, signal builder. RMS acceleration value is calculated from equation 5. In case of set of n values {X1, X2…….Xn}

$$X_{RMS} = \sqrt{\frac{X_1^2 + X_2^2 + \cdots X_n^2}{n}} \tag{5}$$

The Table 2 gives the different types of RMS acceleration values which mean that there is different ride comfort for every input. From the results it can be seen that half sine bump has less value of RMS acceleration so there is more ride comfort. The relation between ride comfort and RMS Acceleration is inversely proportional to each other i.e. when RMS acceleration is more the Ride Comfort is less and vice versa as per the ISO 2631-1:1997.

Table 2: RMS Acceleration for different Road Profiles

S.N.	Road Input	RMS Acceleration (m/s^2)
1	Step Bump	0.813021
2	Half Sine Bump	0.009901
3	Random Source	0.464251
4	Signal Builder	0.488957
5	Sine wave	0.481292
6	Uniform Random	0.613889

VIII. CONCLUSIONS

The half car passive suspension model is analyzed and the mathematical model is developed. And also by using the mathematical model the MATLAB-SIMULINK model is developed. By using both MATLAB SIMULINK and analytical solution i.e. State Space Approach the sprung mass displacement graph is plotted and by both the graphs the model is validated. This validated model is also useful for the different types of road profiles to analyze RMS acceleration.

From the results it can be concluded that the step bump is having more RMS acceleration i.e. 0.813021 m/s^2 and half sine bump is having less RMS acceleration i.e. 0.009901 m/s^2. The relation between ride comfort and RMS Acceleration is inversely proportional to each other i.e. when RMS acceleration is more the Ride Comfort is less and vice versa. Hence by the value of RMS acceleration it can be effectively inferred that the step bump is having lower Ride Comfort compare to half sine bump.

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